Observation of Dynamic Localization in Periodically Curved Waveguide Arrays

S. Longhi, M. Marangoni, M. Lobino, R. Ramponi, and P. Laporta
Dipartimento di Fisica and Istituto di Fotonica e Nanotecnologie del CNR, Politecnico di Milano, Piazza L. da Vinci 32, I-20133 Milano, Italy

E. Cianci and V. Foglietti
Istituto di Fotonica e Nanotecnologie del CNR, sezione di Roma, Via Cineto Romano 42, 00156 Roma, Italy
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We report on a direct experimental observation of dynamic localization (DL) of light in sinusoidally-curved lithium-niobate waveguide arrays which provides the optical analog of DL for electrons in periodic potentials subjected to ac electric fields as originally proposed by Dunlap and Kenkre [Phys. Rev. B **34**, 3625 (1986)]. The theoretical condition for DL in a sinusoidal field is experimentally demonstrated.

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The quantum motion of an electron in a periodic potential subjected to an external field has provided for a long time a paradigmatic model to study fascinating and rather universal coherent dynamical phenomena. These include the long-predicted Bloch oscillations (BO) for dc fields [1], i.e., an oscillatory motion of the wave packet related to the existence of a Wannier-Stark ladder energy spectrum, and the more recently-predicted dynamic localization (DL) for ac fields [2], in which a localized particle periodically returns to its initial state following the periodic change of the field. In recent years, BO have been experimentally observed in a wide variety of systems including semiconductor superlattices [3], atoms in accelerated optical lattices [4], and optical waveguide arrays with a transverse refractive index gradient [5–7]. DL is a phenomenon similar to BO which occurs when the electron is subjected to an ac field. The condition for DL, as originally predicted by Dunlap and Kenkre [2] in the nearest-neighbor tight-binding (NNTB) approximation and for a sinusoidal driving field $E(t) = F\sin(\omega t)$, is that $J_0(\Gamma) = 0$, where $\Gamma = eaF/\hbar\omega$ and a is the lattice period. DL has been shown to be related to the collapse of the quasienergy minibands [8], and the general conditions for DL beyond the NNTB approximation and for generalized ac fields have been identified [9]; DL under the action of both ac and dc fields has been also studied [10], and the influence of excitonic and many-body effects on DL in semiconductor superlattices has been considered (see, e.g. [11,12]). Despite the large amount of theoretical work on DL, experimental evidences of DL are very few [13,14] and much less persuasive than those reported for BO. In semiconductor superlattices, the occurrence of a variety of detrimental effects [12] makes it hard to provide an unambiguous demonstration of DL. The observation of dynamical Bloch band suppression for cold sodium atoms in a frequency-modulated standing light wave has been related to DL [14], however a direct experimental evidence of DL in real space is still lacking. Recently, it has been suggested [15,16] that optical waveguide arrays with a periodically bent axis may provide an ideal laboratory system for an experimental realization of DL in optics, the local curvature of the waveguide providing the optical equivalent of an applied electric field [16,17]. Since optical DL corresponds to a periodic self-imaging of light [16], it bears a close connection with self-collimation and diffraction cancellation effects that have been already demonstrated in photonic crystals (PC) and waveguide arrays [18–20]. In these previous studies, diffractionless propagation is achieved owing to the vanishing of diffraction at the inflection points of the isofrequency PC band surfaces [19,20] or by alternating the sign of diffraction [18] using zigzag waveguides (diffraction management).

In this Letter we provide the first direct experimental observation of DL of light in sinusoidally curved waveguide arrays which exactly mimics the original Dunlap and Kenkre (DK) model [2]. The quantum-optical equivalence between light propagation in sinusoidally curved waveguide arrays and the motion in a crystal of an electron subjected to an ac field has been formally stated in Ref. [16]. The effective two-dimensional (2D) wave equation describing beam propagation in a periodically curved array reads [16]

$$i\bar{\lambda}\frac{\partial\psi}{\partial z} = -\frac{\bar{\lambda}^2}{2n_s}\frac{\partial^2\psi}{\partial x^2} + V(x - x_0(z))\psi, \qquad (1)$$

where z is the paraxial propagation distance, $\bar{\lambda} \equiv \lambda/(2\pi) = 1/k$, $V(x) \simeq n_s - n(x)$, $n(x)$ is the effective refractive index profile of the array with period a [$n(x + a) = n(x)$], n_s is the substrate refractive index, and $x_0(z)$ describes the periodic bending profile of the waveguide with period $\Lambda \gg \lambda$. By means of a Kramers-Henneberger transformation $x' = x - x_0(z)$, $z' = z$, and $\phi(x', z') = \psi(x', z')\exp[-i(n_s/\bar{\lambda})\dot{x}_0(z')x' - i(n_s/2\bar{\lambda})\int_0^{z'} d\xi \dot{x}_0^2(\xi)]$ (where the dot indicates the derivative with respect to z'), Eq. (1) is transformed into the Schrödinger equation for a particle of mass $m = n_s$ and charge q in a periodic potential $V(x')$ under the action of an ac field $\mathcal{E}(z')$:

$$i\lambda\!\!\!^{-} \frac{\partial \phi}{\partial z'} = -\frac{\lambda\!\!\!^{-2}}{2n_s}\frac{\partial^2 \phi}{\partial x'^2} + V(x')\phi - q\mathcal{E}(z')x'\phi, \quad (2)$$

where $\lambda\!\!\!^{-}$ plays the role of the Planck constant and the ac force is related to the axis bending by the equation $q\mathcal{E}(z') = -n_s\ddot{x}_0(z')$. Note that, in the optical analogy, the temporal variable of the quantum problem is mapped into the spatial propagation coordinate z', so that DL is simply observed as a return of the initial light intensity distribution during propagation. For single-mode waveguides, in the NNTB approximation and assuming that the lowest Bloch band of the array is excited, from Eq. (2) the following coupled-mode equations can be derived

$$i\dot{c}_n = -\Delta(c_{n+1} + c_{n-1}) - \frac{qna}{\lambda\!\!\!^{-}}\mathcal{E}(z')c_n, \quad (3)$$

for the amplitudes c_n of the field in the individual waveguides, where $\Delta > 0$ is the coupling constant. The phenomenon of DL [2] corresponds to periodic self-imaging at planes $z = 0, \Lambda, 2\Lambda, 3\Lambda, \ldots$. In the NNTB approximation, the condition for DL is $\int_0^\Lambda d\xi \exp[-i\gamma(\xi)] = 0$, where $\gamma(z) = (An_s a/\lambda\!\!\!^{-})\dot{x}_0(z)$ [9,16]. In particular, for a sinusoidally bent array with period Λ and amplitude A, $x_0(z) = A\sin(2\pi z/\Lambda)$, the condition for DL is [2] $J_0(\Gamma) = 0$, with

$$\Gamma = \frac{4\pi^2 n_s a A}{\Lambda\lambda}. \quad (4)$$

The onset of DL condition $J_0(\Gamma) = 0$ can be at best visualized by a direct measure of the impulse response $[c_n(0) = \delta_{n,0}]$ of a curved array [2,21]. As for the straight array the light spreads linearly with propagation distance z as $|c_n(z)|^2 = |J_n(2\Delta z)|^2$, yielding for the mean square number of excited waveguides the diffusive law $\langle n^2\rangle \equiv \sum_n n^2|c_n|^2 = 2\Delta^2 z^2$, for the sinusoidally curved array, one has instead [2,21]

$$\langle n^2\rangle = 2\Delta^2[u^2(z) + v^2(z)], \quad (5)$$

where we have set $u(z) = \int_0^z d\tau \cos[\mathcal{E}_0\eta(\tau)]$, $v(z) = \int_0^z d\tau \sin[\mathcal{E}_0\eta(\tau)]$, $\mathcal{E}_0 = 4\pi^2 a n_s A/(\lambda\!\!\!^{-}\Lambda^2)$, and $\eta(z) = [1 - \cos(2\pi z/\Lambda)]/[2\pi/\Lambda]$. In particular, for a modulation period Λ smaller than the coupling length $1/\Delta$, one has $|c_n(z)|^2 \simeq |J_n(2\Delta J_0(\Gamma)z)|^2$ and $\langle n^2\rangle \simeq 2\Delta^2 J_0^2(\Gamma)z^2$. Therefore, when the condition $J_0(\Gamma) = 0$ is satisfied, an effective suppression of waveguide coupling is attained, which corresponds to a coherent destruction of tunneling [21]. For $J_0(\Gamma) \neq 0$, the light diffraction pattern is analogous to that of a straight array with an effective coupling coefficient $\Delta_{\rm eff} = \Delta|J_0(\Gamma)|$. In our experiments, we could vary the parameter Γ [Eq. (4)] by either tuning the probing wavelength λ or by measuring, for a given wavelength, the diffraction patterns of several arrays with different modulation period Λ. The first realized sample (Fig. 1) consists of a set of one straight and two sinusoidally curved arrays of waveguides with a graded index profile fabricated by the annealed proton exchange (APE) technique in z-cut congruent lithium niobate [22]. The channel width ($\simeq$7 μm) and the fabrication parameters have been chosen such that the individual waveguide turns out to be single mode, for TM polarization, in the whole spectral range from $\lambda = 1440$ nm to $\lambda = 1610$, with an upper cutoff wavelength at around $\lambda \simeq 1660$ nm. Each array is made by a set of 80 identical and equally spaced $L = 28$ mm-long waveguides with separation $a = 14$ μm. Array (2) comprises 7 sinusoidal cycles ($\Lambda = 4$ mm and $A \simeq 13$ μm), with $\Gamma \simeq 2.405$ at $\lambda \simeq 1610$ nm, whereas array (3) comprises only a semicycle ($\Lambda = 56$ mm and $A \simeq 164$ μm), with $\Gamma \simeq 2.405$ at $\lambda \simeq 1440$ nm. The waveguide coupling constant Δ has been measured as a function of the probing wavelength λ by the analysis of the diffraction pattern obtained for the straight array (1) with a single waveguide excitation at the input plane (Fig. 2). Light coupling was accomplished by focusing the circular diffraction-limited beam of a tunable semiconductor laser (Agilent Mod. 81600B) into the input face of one waveguide of the array to obtain a focused spot size of radius $\simeq 3.5$ μm. The light beam distribution at the exit of the array was imaged and recorded onto an IR Vidicon camera. The measured

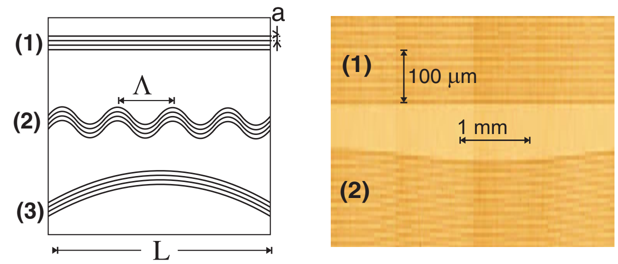

FIG. 1 (color online). Left: Schematic of the first manufactured sample showing the three waveguide arrays: the straight array (1), the sinusoidally curved array with short period (2), and the semicycle sinusoidal array (3). Right: Microscope image showing a particular of the straight and short-period curved arrays.

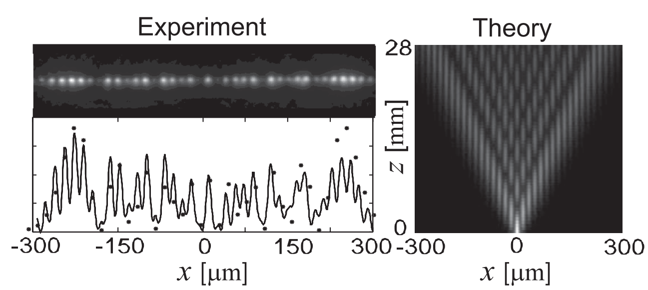

FIG. 2. Left: 2D output intensity pattern recorded on the IR camera for single waveguide excitation at $\lambda = 1610$ nm of the straight array, and corresponding intensity profile cross section. The points show the waveguide intensity distribution $|c_n|^2 = |J_n(2\Delta L)|^2$ predicted by the coupled-mode Eqs. (3) for $\Delta = 3$ cm^{-1}. Right: beam evolution (top view) predicted by numerical simulations for the straight array.

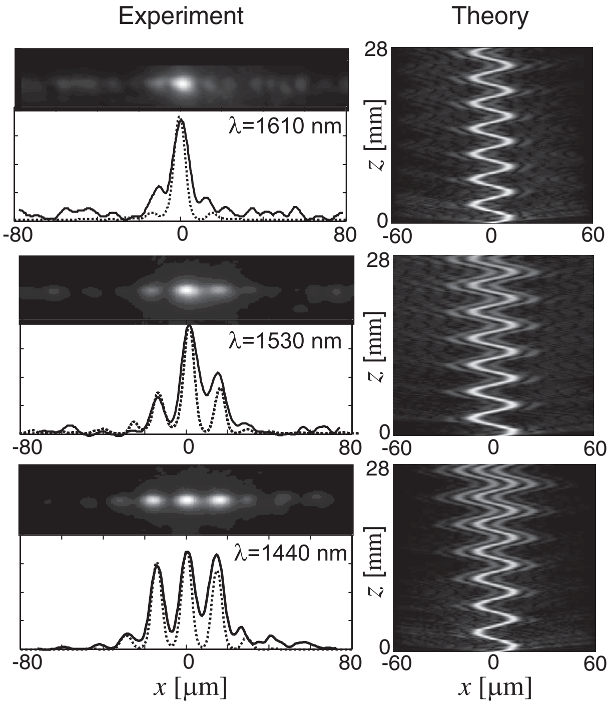

FIG. 3. Left: 2D output intensity patterns recorded on the IR camera for a single waveguide excitation of the short-period curved array at different wavelengths, and corresponding intensity profile cross sections (solid curves). In the figures the dashed curves are the numerically predicted intensity profiles. Right: corresponding beam evolution (top view) predicted by numerical simulations of Eq. (1).

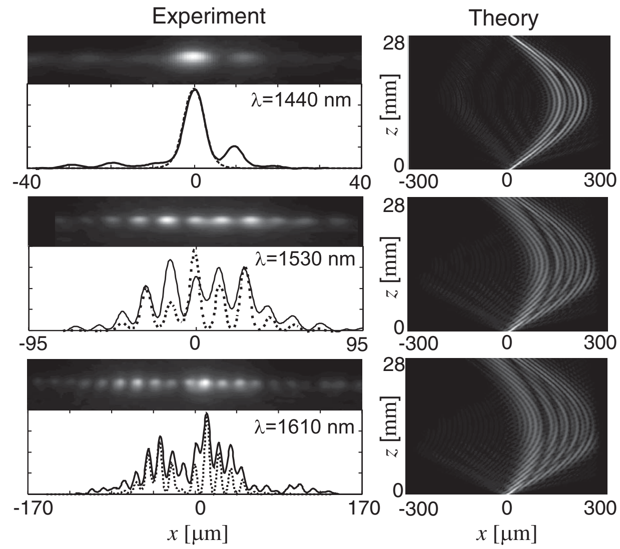

FIG. 4. Same as Fig. 3, but for the semicycle curved array.

coupling constant Δ increases as the wavelength is increased and varies from $\Delta \sim 1.75\ \mathrm{cm}^{-1}$ at $\lambda = 1440$ nm to $\Delta \sim 3\ \mathrm{cm}^{-1}$ at $\lambda = 1610$ nm (the case shown in Fig. 2). The measured impulse response of the curved array (2) for a few values of λ is shown in Fig. 3, together with the beam evolution along the arrays as predicted by a numerical analysis of Eq. (1). Note that, as the wavelength λ is tuned at the $\Gamma = 2.405$ value (top figure) a single spot is observed at the output plane, whereas far from the DL condition the number of excited waveguides increases. Note however that the change of Γ around $\sim$2.4 allowed by wavelength tuning is rather modest (about 10%), which explains why the number of excited output waveguides is relatively small ($\sim$ 3–4) even at $\lambda = 1440$ nm. An important property of DL with a sinusoidal ac modulation is that wave refocusing is attained not only at multiples of the modulation cycle as shown for array (2) in Fig. 3, but also at multiples of the semicycle. This is clearly demonstrated both numerically and experimentally in Fig. 4, which shows the impulse response of array (3). We note that, in the context of DL, the refocusing property in a semicycle was theoretically investigated for an ac square-wave driving field [15,23] and related to periodic BO motion with alternating sign for the dc field (the so-called ac BO [15]). However, the dynamics shown in Fig. 4 is not a BO motion with a dc field, which would strictly require circularly curved waveguides [15,17].

We also checked the onset of DL by broad beam excitation illuminating at normal incidence the input facet of the array with an astigmatic Gaussian beam of varying width w_x in the x direction. As an example, Fig. 5 shows diffraction cancellation observed in array (3) with excitation at $\lambda = 1440$ nm for two values of w_x; similar results are obtained for array (2). It should be noted that an analogous diffraction suppression was previously observed in Ref. [18] using zigzag arrays (see, in particular, Figs. 3 and 4). In that case, diffraction cancellation arises from the alternation of positive and negative diffraction, obtained by periodic tilting of straight waveguide pieces. This is basically the same idea of ‘‘dispersion management’’ known for temporal pulse propagation in dispersive media. This scheme of diffraction cancellation, however, cannot be regarded as a realization of DL, i.e., wave localization induced by a periodic ac force, as intended within the original DK model (sinusoidal ac field) or its generalizations [9], including a square-wave ac field [15]. In fact, for zigzag pieces of straight waveguides the curvature $\ddot{x}_0$ is

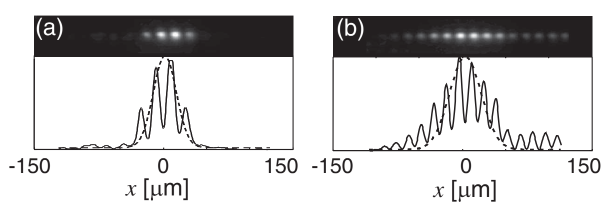

FIG. 5. Measured 2D output intensity patterns and corresponding cross sections for an astigmatic Gaussian beam excitation of array (3) with (a) $w_x \sim 24.7\ \mu$m, and (b) $w_x \sim 37.4\ \mu$m. The dashed curves are the input Gaussian beam profiles in the x direction.

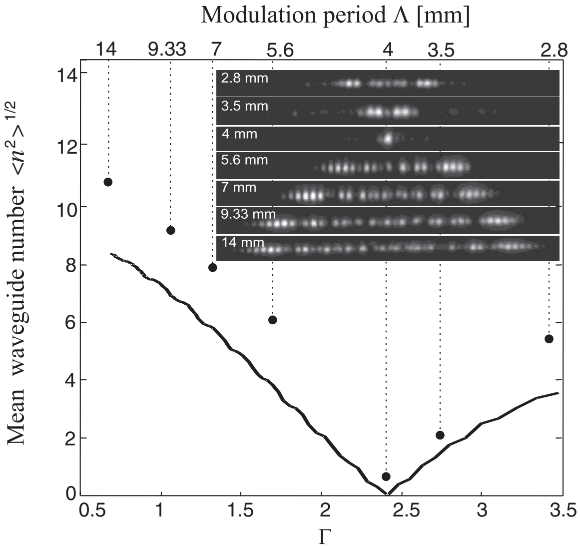

FIG. 6. Measured number of excited waveguides $\sqrt{\langle n^2 \rangle}$ versus Γ (dots) for 7 waveguide arrays with increasing values of Λ, and corresponding theoretical behavior (solid curve). The recorded 2D output intensity profiles are also shown.

zero and the $\mathcal{E}$ field vanishes in the quantum-mechanical model [see Eq. (2)]. At the tilting planes of the waveguide axis, however, the field wave vector (momentum) experiences an instantaneous change of its direction with respect to the waveguide axis with a corresponding change of the diffraction sign, leading to a zero mean-diffraction coefficient according to Eq. (4) of Ref. [18]. In the quantum-mechanical model (2), this would correspond to a quantum particle moving in a periodic potential whose momentum is abruptly and periodically kicked.

To quantitatively check the diffusive law (5), we fabricated a second sample consisting of 7 sinusoidally curved arrays with the same length and design parameters as those of the array (2) in Fig. 1 but with different and increasing values of Λ [from 2.8 to 14 mm], enabling to span a wide range of Γ values not achievable by simple wavelength tuning. The impulse response of the 7 arrays, excited at $\lambda = 1610$ nm, was recorded, and the mean square number of excited waveguides $\langle n^2 \rangle$ was estimated from the measured cross-sectional profiles $|\psi(x, L)|^2$ by the relation $\langle n^2 \rangle \sim \int dx (x/a)^2 |\psi(x, L)|^2$. The experimental results are summarized in Fig. 6 and compared to the theoretical prediction [Eq. (5)]. Note that the spreading of the output patterns as Γ departs from 2.405 is related to an increase of the effective coupling coefficient $\Delta_{\text{eff}} = \Delta |J_0(\Gamma)|$, which plays an analogous role of the mean-diffraction coefficient in the diffraction management scheme of Ref. [18] (compare Fig. 6 with Fig. 4 of Ref. [18]). The occurrence of the minimum in Fig. 6 is thus a clear signature of DL. In conclusion, we have experimentally observed DL of photons in sinusoidally curved waveguide arrays, a phenomenon which mimics the quantum-mechanical effect originally proposed by Dunlap and Kenkre [2]. In particular, the basic condition for DL in a sinusoidal field, $J_0(\Gamma) = 0$, has been experimentally confirmed.

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